

INDIAN SCHOOL MUSCAT

FIRST PRE-BOARD EXAMINATION
JANUARY 2021

SET A

CLASS XII

Marking Scheme – SUBJECT [THEORY]

Q.NO.	Answers	Marks (with split up)
1.	0	
2.	4	
3.	{(1,1) , (2,2) , (3,3) , (1,2) ,(2,1) , (2,3) , (3,2) }	
4.	$x = 3$	
5.	81	
6.	405	
7.	1 OR 3/2	
8.	25/4 OR 16/3	
9.	4,2 OR sec x	
10.	2/3	
11.	8/7	
12.	45 degree	
13.	(2,3,-5)	
14.	$\sqrt{17}$	
15.	1	
16.	20	
17.	i) C ii) b iii) b iv) b v) c i) A ii) b iii) d iv) d v) a	
	19) $\sin^{-1}(\cos 45 \sin 20 + \sin 45 \cos 20)$ $\sin^{-1}(\sin 65)$ $= 65$	1 1
20)	$A^2 = \begin{pmatrix} 16 & 21 \\ 28 & 37 \end{pmatrix}$ $A^{-1} = \begin{pmatrix} -5/2 & 3/2 \\ 2 & -1 \end{pmatrix}$	1+1
21)	$P = \frac{1}{2}$ OR $a = 4 : b = -4$	1+1
22)	Slope of chord = -1/2 $2(x-3) = -1/2$ $(11/4 \quad 1/16)$	$\frac{1}{2}$ 1 $\frac{1}{2}$
	23) Multiply Nr and Denominator by Cosecx + cotx $= \log (\text{cosecx} - \cot x) + c$ OR $\cot x = \cos x / \sin x$ Simplify $\frac{1}{2}(x - \log(\sin x + \cos x)) + c$	1 $\frac{1}{2} + \frac{1}{2}$
24)	$2 \int_0^{\frac{\pi}{2}} \cos x \, dx$ $= 2 \sin x]$ $= 2 \text{ sq.units}$	1 $\frac{1}{2}$ $\frac{1}{2}$

	25) $\frac{dy}{dx} = e^{ax} \cdot e^{bx}$ $\int \frac{dy}{e^{by}} = \int e^{ax} dx$ $\frac{e^{ax}}{a} + \frac{e^{-bx}}{b} = c$	$\frac{1}{2}$ 1 $\frac{1}{2}$
	26) $\frac{1}{2} \widehat{\vec{d}_1} \times \widehat{\vec{d}_2} = 5\sqrt{3}$ sq.units	
	27) $\frac{x-1}{-3} = \frac{y-2}{\frac{2}{7}p} = \frac{z-2}{2}$ $\frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$ $\widehat{\vec{b}_1} \cdot \widehat{\vec{b}_2} = 0$ $p = 70/11$	1 $\frac{1}{2}$ $\frac{1}{2}$
	28) $0.4 \times 0.4 \times 0.4 \times 0.4 = 0.0256$ or $12/15 * 11/14 * 10/13 = 44/91$	1+1
	29) Reflexive, Symmetric and Transitive $(1,1), (1,4), (1,7), (1,10) \in R$ $[1] = \{1,4,7,10\}$	2 1
	30) $u = (\sin x)^x$ $v = \sin(x^x)$ $du/dx = (\sin x)^x [x \cot x + \log \sin x]$ $dv/dx = x^x \cos(x^x)(1 + \log x)$	$\frac{1}{2}$ 1+1 $\frac{1}{2}$
	OR) $dx/dt = \frac{a \cos^2 t}{\sin t}$ $dy/dt = a \cdot \cos t$ $dy/dx = a \cdot \cos t \cdot \frac{\sin t}{a \cos^2 t} = \tan t$	
	31) $28 \log x + 17 \log y = 45 \log(x+y)$ $\frac{28}{x} + \frac{17}{y} y' = \frac{45}{x+y} (1 + y')$ $y' = \frac{y}{x}$	1 1 1
	32) $f'(x) = \frac{\cos x - \sin x}{2 + \sin 2x}$ $2 + \sin x > 0$ $\tan x < 1$ $f'(x) > 0 \text{ in } (0, \frac{\pi}{4})$ f is strictly increasing in $(0, \frac{\pi}{4})$	2 $\frac{1}{2}$ $\frac{1}{2}$
	33) $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$ $t = \sin x - \cos x: dt = (\sin x + \cos x)dx$ $\int_{-1}^0 \frac{dt}{25 - 16t^2}$ $1/40 \log 9$	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$
	34) Area = $2 \int_0^4 \sqrt{16 - x^2} dx$ $2(\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1}(\frac{x}{4}))$ 8π sq.units OR Area = $2 \int_0^{\sqrt{5}} (5 - x^2) dx$	1 1 $\frac{1}{2} + \frac{1}{2}$

	$2(5x \frac{x^3}{3})$ $20\frac{\sqrt{5}}{3}$ sq.units	
	35) $p = \sec^2 x$ $Q = \tan x \sec^2 x$ IF = $e^{\tan x}$ Solution : $y e^{\tan x} = \int t e^t dt$ $y = \tan x - 1 + e/e^{\tan x}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$
	36) $5x+4y+3z = 11000$ $4x+3y+5z = 10700$ $x + y + z = 2700$ $AX = B$ $ A = -3$ $\text{adj } A = \begin{pmatrix} -2 & -1 & 11 \\ 1 & 2 & -13 \\ 1 & -1 & -1 \end{pmatrix}$ $x = 1000, y = 900, z = 800$ OR $A' = \begin{pmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & 15 \end{pmatrix}$ $(AB)^{-1} = \begin{pmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & 42 \end{pmatrix}$	1 1 1 1
	37) $x = 2\lambda + 7, y = 4\lambda + 14, z = -\lambda + 5$ $\lambda = -3$ Foot of perpendicular (1,2,8) Length = $3\sqrt{21}$ Image = (-5,-10,11) OR P(-6, -4, -2) $AP = \sqrt{149}$ units	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$
	38) corner points A (60,0) B (120, 0) C(60,30) D (40,20) Minimum = 300 at (60,0) Maximum = 600 at all points on the line segment joining the points (120,0) and (60,30) OR MINIMUM = 1550 At (0,5)	3 + 2
	1){(1,1), (1,2), (2,1)} 2) 2 3) 0 4) $K = 3$ 5) 288 6) 256 7) -1 OR $\frac{4}{5}$ 8) $25/4$ OR $16/3$ 9) 3 OR $e^{2\sqrt{x}}$ 10) $4/3$	SET B

	11) $\sqrt{65}$ 12) $\sqrt{94}/2$ 13) (-2,3,5) 14) 8/7 15) 1 16) k 17) c,b,b,a,c 18) a,b,d,d,a	
	19) $\tan^2(\tan^{-1}(\sqrt{3}) + \cot^2(\cot^{-1} 2\sqrt{2}))$ $3 + 8 = 11$	
	20) $A^{-1} = 5I - A / 7$ $\begin{pmatrix} 2/7 & -1/7 \\ 1/7 & 3/7 \end{pmatrix}$ <p>OR</p> $P = \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix}; Q = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$	
	21) LHL = RHL = $f(\frac{\pi}{2})$ $a = \frac{\pi}{6}, b = -\frac{\pi}{12}$ <p>OR</p> $a = 6 \text{ and } b = -9$	
	22) SET A	
	23) TB PAGE 302 $= \log \sec x + \tan x + C$ <p>OR</p> $= \frac{1}{2} [x + \log \sin x + \cos x] + C$	
	24) $\int_0^2 (x+2) dx$ $= 6 \text{ sq.units}$	
	27) $\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$ $\hat{b}_1 \times \hat{b}_1 = 3\hat{i} - \hat{j} - 7\hat{k}$ $ \hat{b}_1 \times \hat{b}_1 = \sqrt{59}$ $SD = 10/\sqrt{59}$	
	28) $P(\text{problem solved}) = 1 - P(\text{not solved})$ $= 1 - (1 - 1/3)(1 - 1/4)(1 - 1/5)$ $= 1 - 2/5 \cdot 3/4 \cdot 4/5$ $= 3/5$ <p>OR</p> $P(E \cap F) = 43/1000$ $P(F) = 430/1000$ $P(E/F) = 1/10$	
	29) Reflexive, Symmetric and Transitive $[3] = \{1, 3, 5, 7, 9\}$	
	30) $y \log \sin x = x \log \sin y$	

Diff. both sides w.r.t x

$$\frac{dy}{dx} = \frac{\log \sin y - y \cot x}{\log \sin x - x \cot y}$$

OR

$$\frac{dy}{dt} = a^{(t+\frac{1}{t})} \cdot \log a \cdot \left(1 - \frac{1}{t^2}\right)$$

$$\frac{dx}{dt} = a \left(t + \frac{1}{t}\right)^{a-1} \cdot \left(1 - \frac{1}{t^2}\right)$$

$$\frac{dy}{dx} = \frac{a^{t+\frac{1}{t}} \cdot \log a \left(1 - \frac{1}{t^2}\right)}{a \left(t + \frac{1}{t}\right)^{a-1} \left(1 - \frac{1}{t^2}\right)} = \frac{dy}{dx} = \frac{a^{t+\frac{1}{t}} \cdot \log a}{a \left(t + \frac{1}{t}\right)^{a-1}}$$

33) TB Page 351 , example 44

$$I = \int_0^\pi \frac{\pi - x}{a^2[\cos(\pi - x)]^2 + b^2[\sin(\pi - x)]^2} dx$$

$$2I = \int_0^\pi \frac{\pi}{a^2 \cos^2 x + b^2 \sin^2 x} dx$$

$$I = \frac{\pi}{2} \int_0^\pi \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$$

$$I = \frac{\pi}{b} \int_0^\infty \frac{dt}{a^2 + t^2}$$

$$I = \frac{\pi^2}{2ab}$$

$$34) 2x \int_0^5 \sqrt{25 - x^2} dx$$

$25\pi/2$ sq.units

OR

$$2x \int_0^2 (x^2 - 4) dx$$

= $32/3$ sq.units

$$36) x + y + z = 10$$

$$2x + y = 13$$

$$x + y - 4z = 0$$

$$|A| = 5$$

$$\text{adj } A = \begin{pmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -4 \end{pmatrix}$$

$$A^{-1} = 1/5 \begin{pmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -4 \end{pmatrix}$$

$$X = 1/5 \begin{pmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -4 \end{pmatrix} \begin{pmatrix} 10 \\ 13 \\ 0 \end{pmatrix}$$

$$x = 5, y = 3, z = 2$$

OR

$$AB = -BA$$

$$\begin{pmatrix} a-b & 0 \\ 2a-b & -1 \end{pmatrix} = \begin{pmatrix} -a+2 & a-1 \\ -b+2 & b-1 \end{pmatrix}$$

	a = 1 ; b = 0	
	<p>Eqn of plane is A (x-1) + B (y-2) + C (z+1) = 0 (2,0,2) is a point on the plane A -2B +3C = 0</p> <p>$\vec{b} \cdot \vec{n} = 0$ A + 2B +2C = 0 Eqn. of plane is 10 x - y - 4z = 12</p> <p>OR</p> <p>$3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4) = 0$ $\lambda = -1$ Foot of Perpendicular (3,5,9) Eqn. of plane through (1,2,3), (3,5,9) and (6,7,7)</p> $\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 6 \\ 5 & 5 & 4 \end{vmatrix} = 0$ $18x + 22y + 5z = 77$	
	<p>1) 3 OR x</p> <p>2) 25/4 or 16/3</p> <p>3) -2 or 4/5</p> <p>4) 135</p> <p>5) 4^9</p> <p>6) 4</p> <p>7) $\{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$</p> <p>8) 18</p> <p>9) (2,-3,5)</p> <p>10) $5\sqrt{2}$</p> <p>11) $\frac{1}{4}$</p> <p>12) 10/13</p> <p>13) $\sqrt{94}/2$</p> <p>14) 1</p> <p>15) $\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$</p> <p>16) 0</p> <p>17) a,b,d,b,d</p> <p>18) a,b,d,d,a</p> <p>19) 70</p>	SET C
	<p>22) $\int \tan^4 x \, dx$</p> $\int \tan^2 x \cdot \tan^2 x \, dx$ $\int \tan^2 x \cdot (\sec^2 x - 1) \, dx$ $= \frac{\tan^3 x}{3} - \tan x + x + c$	
	23) k = 1/2	
	29) { 1,5,9 }	

$$f'(x) = \frac{\cos x (4 - \cos x)}{(2 + \cos x)^2}$$

35)

Putting $f'(x) = 0$

$$\frac{\cos x (4 - \cos x)}{(2 + \cos x)^2} = 0$$

$$\therefore \cos x (4 - \cos x) = 0$$

Thus, we divide the interval $(0, 2\pi)$ into three disjoint intervals

$$(0, \frac{\pi}{2}), (\frac{\pi}{2}, \frac{3\pi}{2}) \text{ & } (\frac{3\pi}{2}, 2\pi)$$

Value of x	Interval	Sign of $f'(x) = \frac{\cos x (4 - \cos x)}{(2 + \cos x)^2}$	Nature of $f(x)$
$0 < x < \frac{\pi}{2}$	$x \in (0, \frac{\pi}{2})$	$\frac{(+)(+)}{(+)} = (+)$	Strictly Increasing
$\frac{\pi}{2} < x < \frac{3\pi}{2}$	$x \in (\frac{\pi}{2}, \frac{3\pi}{2})$	$\frac{(-)(+)}{(+)} = (-)$	Strictly Decreasing
$\frac{3\pi}{2} < x < 2\pi$	$x \in (\frac{3\pi}{2}, 2\pi)$	$\frac{(+)(+)}{(+)} = (+)$	Strictly Increasing

$f(x)$ is strictly increasing on $(0, \frac{\pi}{2}) \text{ & } (\frac{3\pi}{2}, 2\pi)$

$f(x)$ is strictly decreasing on $(\frac{\pi}{2}, \frac{3\pi}{2})$